

Imatest - Colorcheck Appendix

Algorithms and reference formulas

This page contains algorithms and reference formulas for [Colorcheck](#) and [Multicharts](#). It's all in **green text** because it's all math.



Color difference (error) formulas

The notation on this page is adapted from the [Digital Color Imaging Handbook](#), edited by Gaurav Sharma, published by the CRC Press, referred to below as DCIH. The DCIH [online Errata](#) was consulted.

In measuring color error, keep it in mind that **accurate** color is not necessarily the same as **pleasing** color. Many manufacturers deliberately alter colors to make them more pleasing, most often by increasing saturation. (That is why [Fujichrome Velvia](#) was so successful when it was introduced in the 1990.) In calculating color error, you may choose not to use the exact ColorChecker (or other chart) $L^*a^*b^*$ reference values; you may want to substitute your own enhanced values. Imatest Master allows you to enter values from a file written in CSV format (you can save the values from a measured chart using [Multicharts](#)).

Imatest users frequently ask about the meaning of “corr” and “uncorr” in the Colorcheck a^*b^* error plot. Corr means that the mean saturation (mean chroma; $\text{mean}(\sqrt{a^{*2}+b^{*2}})$) of the camera is adjusted to be the same as that of the reference before making the comparison. This is a very easy correction to make; it tells how accurate the color could be if the mean chroma were the same as the reference. See [below](#) for more detail.

Absolute differences (including luminance)

CIE 1976

The $L^*a^*b^*$ color space was designed to be **relatively** perceptually uniform. That means that perceptible color difference is approximately equal to the Euclidean distance between $L^*a^*b^*$ values. For colors $\{L_1^*, a_1^*, b_1^*\}$ and $\{L_2^*, a_2^*, b_2^*\}$, where $\Delta L^* = L_2^* - L_1^*$, $\Delta a^* = a_2^* - a_1^*$, and $\Delta b^* = b_2^* -$

b_1^* ,

$$\Delta E_{ab}^* = ((\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2)^{1/2} \quad (\text{DCIH (1.42, 5.35)}; \dots)^{1/2} \text{ denotes square root of } (\dots).$$

Although ΔE_{ab}^* is relatively simple to calculate and understand, it's not very accurate especially for strongly saturated colors. $L^*a^*b^*$ is not as perceptually uniform as its designers intended. For example, for saturated colors, which have large chroma values ($C^* = (a^{*2} + b^{*2})^{1/2}$), the eye is less sensitive to changes in chroma than to corresponding changes for Hue ($\Delta H^* = ((\Delta E_{ab}^*)^2 - (\Delta L^*)^2 - (\Delta C^*)^2)^{1/2}$) or Luminance (ΔL^*). To address this issue, several additional color difference formulas have been established. In these formulas, just-noticeable differences (JNDs) are represented by *ellipsoids* rather than circles.

CIE 1994

The CIE-94 color difference formula, ΔE_{94}^* , provides a better measure of perceived color difference than ΔE_{ab}^* .

$$\Delta E_{94}^* = ((\Delta L^*)^2 + (\Delta C^*/S_C)^2 + (\Delta H^*/S_H)^2)^{1/2} \quad (\text{DCIH (5.37)}; \text{ omitting constants set to 1}), \text{ where}$$

$$S_C = 1 + 0.045 C^*; \quad S_H = 1 + 0.015 C^* \quad (\text{DCIH (1.53, 1.54)})$$

[$C^* = ((a_1^{*2} + b_1^{*2})^{1/2} (a_2^{*2} + b_2^{*2})^{1/2})^{1/2}$ (the geometrical mean chroma) gives symmetrical results for colors 1 and 2. However, when one of the colors (denoted by subscript s) is the standard, the chroma of the standard, $C_s^* = (a_s^{*2} + b_s^{*2})^{1/2}$, is preferred for calculating S_C and S_H . The asymmetrical equation is used by Bruce Lindbloom.]

$$\Delta H^* = ((\Delta E_{ab}^*)^2 - (\Delta L^*)^2 - (\Delta C^*)^2)^{1/2} \quad (\text{hue difference ; DCIH (5.36)})$$

$$\Delta C^* = (a_1^{*2} + b_1^{*2})^{1/2} - (a_2^{*2} + b_2^{*2})^{1/2} \quad (\text{chroma difference})$$

CMC

The CMC color difference formula is widely used by the textile industry to match bolts of cloth. Although it's less familiar to photographers than the CIE 1976 geometric distance ΔE^*_{ab} , it was one of the best measurement metrics prior to CIEDE2000. But it never gained traction in the photographic industry. It is slightly asymmetrical: subscript s denotes the standard (reference) measurement. CMC is the [Color Measurement Committee](#) of the [Society of Dyers and Colourists](#) (UK).



$$\Delta E^*_{CMC}(l,c) = ((\Delta L^*/S_L)^2 + (\Delta C^*/S_C)^2 + (\Delta H^*/S_H)^2)^{1/2} \quad (DCIH (5.37)), \text{ where}$$

(That's the lowercase letter *l* in (l,c) and the denominator of $(\Delta L^*/S_L)^2$.) $\Delta E^*_{CMC}(1,1)$ ($l = c = 1$) is used for graphic arts perceptibility measurements. $l = 2$ is used in the textile industry for acceptability of fabric matching. For now I mate test displays $\Delta E^*_{CMC}(1,1)$.

$$S_L = 0.040975 L_s^* / (1 + 0.01765 L_s^*); \quad L_s^* \geq 16 \quad (DCIH (1.48))$$

$$= 0.511; \quad L_s^* < 16$$

$$S_C = 0.0638 C_s^* / (1 + 0.0131 C_s^*) + 0.638; \quad S_H = S_C (T_{CMC} F_{CMC} + 1 - F_{CMC}) \quad (DCIH (1.49, 1.50))$$

$$F_{CMC} = ((C_s^*)^4 / ((C_s^*)^4 + 1900))^{1/2} \quad (DCIH (1.51));$$

$$T_{CMC} = 0.56 + | 0.2 \cos(h_s^* + 168^\circ) | \quad 164^\circ \leq h_s^* \leq 345^\circ \quad (DCIH (1.52))$$

$$= 0.36 + | 0.4 \cos(h_s^* + 35^\circ) | \quad \text{otherwise}$$

ΔH^* and ΔC^* have the same formulas as CIE-94.

CIEDE2000

The CIEDE2000 formulas (ΔE_{00} and ΔC_{00}) are the upcoming standard, and may be regarded as more accurate than the previous formulas. We omit the equations here because they are described very well on Gaurav Sharma's [CIEDE2000 Color-Difference Formula](#) web page. Default values of 1 are used for parameters k_L , k_C , and k_H .

At the time of this writing (February 2008) the CIE 1976 color difference metrics ($\Delta E^*_{ab} \dots$) are still the most familiar. CIE 1994 is more accurate and robust, and retains a relatively simple equation. ΔE^*_{CMC} is more complex but widely used in the textile industry. The complexity of the [CIEDE2000 equations](#) (DClH, section 1.7.4, pp. 34-40) has slowed their widespread adoption, but they are on their way to becoming the accepted standard. **For the long run, CIEDE2000 color difference metrics are the best choice.**

Color differences that omit luminance difference

Since Colorcheck measures captured images, exposure errors will strongly affect color differences ΔE^*_{ab} , ΔE^*_{94} , and ΔE^*_{00} . Since it is useful to look at color errors independently of exposure error, we define color differences that omit ΔL^* .

$$\Delta C_{ab} = ((\Delta a^*)^2 + (\Delta b^*)^2)^{1/2}$$

$$= ((\Delta E^*_{ab})^2 - (\Delta L^*)^2)^{1/2} \quad (\text{This is a more general form: Delta-E with Delta-L removed.})$$

$$\Delta C_{94} = ((\Delta C^*/S_C)^2 + (\Delta H^*/S_H)^2)^{1/2}$$

$$\Delta C_{CMC} = ((\Delta C^*/S_C)^2 + (\Delta H^*/S_H)^2)^{1/2}$$

$$\Delta C_{00} \text{ omits the } (\Delta L^*/k_L S_L)^2 \text{ term from the } \Delta E_{00} \text{ (square root) equation (see } \a href="#">Sharma).$$

These formulas don't entirely remove the effects of exposure error since a^* and b^* are affected somewhat by exposure, but they reduce it to a manageable level. Note that the ΔC definition can cause some confusion because it is different from the pure chroma difference, ΔC^* , where chroma = $C^* = \sqrt{a^{*2} + b^{*2}}$. ΔC^* is not suitable as a perceptual measurement because it does not include hue.

Notation changes (made for consistency with textbooks such as DClH)

$\Delta C_{ab} = ((\Delta a^*)^2 + (\Delta b^*)^2)^{1/2}$, formerly called $\Delta E(a^*b^*)$, is the color difference (only), with L^* omitted.

$\Delta E^*_{ab} = ((\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2)^{1/2}$, formerly called $\Delta E(L^*a^*b^*)$, is the total difference, including L^* .

This notation is somewhat confusing because ΔE^*_{ab} includes L^* (as well as a and b) in its formula.

Color differences corrected for chroma (saturation) boost/cut

Many digital cameras deliberately boost chroma, i.e., saturation, to enhance image appearance in digital cameras. This boost increases color error in the ΔE^* and ΔC formulas, above.

The mean chroma percentage is

$$Chrp = 100\% * (\text{measured mean}((a_i^{*2} + b_i^{*2})^{1/2}) / (\text{Colorchecker mean}((a_i^{*2} + b_i^{*2})^{1/2})))$$

$$= 100\% * \text{mean}(C_{\text{measured}}) / \text{mean}(C_{\text{ideal}}); \quad C_i = (a_i^{*2} + b_i^{*2})^{1/2}$$

$$i \leq 1 \leq 18 \quad (\text{the first three rows of the Colorchecker})$$

Chroma, which is closely related to the perception of saturation, is boosted when $Chrp > 100$. Chroma boost increases color error measurements ΔE^*_{ab} , ΔC^*_{ab} , ΔE^*_{94} , and ΔC^*_{94} . Since it is easy to remove chroma boost in image editors (with saturation settings), it is useful to measure the color error after the mean chroma has been corrected (normalized) to 100%. To do so, normalized a_{i_corr} and b_{i_corr} are substituted for measured (camera) values a_i and b_i in the above equations.

$$a_{i_corr} = 100 a_i / Chrp; \quad b_{i_corr} = 100 b_i / Chrp$$

The reference values for the ColorChecker are unchanged. Color differences corrected for chroma are denoted $\Delta C^*_{ab}(\text{corr})$, $\Delta C^*_{94}(\text{corr})$, and $\Delta C^*_{CMC}(\text{corr})$.

Mean and RMS values

Colorcheck Figure 3 reports the mean and RMS values of ΔC^*_{ab} corrected (for saturation) and uncorrected, where

$mean(x) = \sum x_i / n$ for n values of x .

$RMS(x) = \sigma(x) = (\sum x_i^2 / n)^{1/2}$ for n values of x .

The RMS value is of interest because it gives more weight to the larger errors.

Algorithm

- Locate the regions of interest (ROIs) for the 24 ColorChecker zones.
- Calculate statistics for the six grayscale patches in the bottom row, including the average pixel levels and a second order polynomial fit to the pixel levels in the ROIs— this fit is subtracted from the pixel levels for calculating noise. It removes the effects of nonuniform illumination. Calculate the noise for each patch.
- Using the average pixel values of grayscale zones 2-5 in the bottom (omitting the extremes: white and black), the average pixel response is fit to a mathematical function (actually, two functions). This requires some explanation.

A simplified equation for a capture device (camera or scanner) response is,

$normalized\ pixel\ level = (pixel\ level / 255) = k_1 \text{ exposure}^{gamc}$

*Gamc is the **gamma** of the capture device. Monitors also have gamma = gamm defined by*

$monitor\ luminance = (pixel\ level / 255)^{gamm}$

Both gammas affect the final image contrast,

$System\ gamma = gamc * gamm$

Gamc is typically around $0.5 = 1/2$ for digital cameras. Gamm is 1.8 for Macintosh systems; gamm is 2.2 for Windows systems and several well known color spaces (sRGB, Adobe RGB 1998, etc.). Images tend to look best when system gamma is somewhat larger than 1.0, though this doesn't always hold— certainly not for contrasty scenes. For more on gamma, see [Glossary](#), [Using Imatest SFR](#), and [Monitor calibration](#).

*Using the equation, **$density = -\log_{10}(exposure) + k$** ,*

$$\log_{10}(\text{normalized pixel level}) = \log_{10}(k_1 \text{ exposure}^{\text{gamc}}) = k_2 - \text{gamc} * \text{density}$$

This is a nice first order equation with slope *gamc*, represented by the **blue** dashed curves in the figure. But it's not very accurate. A second order equation works much better:

$$\log_{10}(\text{normalized pixel level}) = k_3 + k_4 * \text{density} + k_5 * \text{density}^2$$

k_3 , k_4 , and k_5 are found using second order regression and plotted in the **green** dashed curves. The second order fit works extremely well.

- Saturation *S* in HSV color representation is defined as $S(\text{HSV}) = (\max(R,G,B) - \min(R,G,B)) / \max(R,G,B)$. *S* correlates more closely with perceptual White Balance error in HSV representation than it does in HSL.

The the equation for saturation boost in the lower image of the third figure is $S' = (1 - e^{-4S}) / (1 - e^{-4})$, where $e = 2.71828...$

Grayscale levels and exposure error

The Colorchecker grayscale patch densities (in the bottom row) are specified as 0.05, 0.23, 0.44, 0.70, 1.05, and 1.50. Using the equation, $\text{pixel level} = 255 * (10^{-\text{density}} / 1.06)^{(1/2.2)}$ (see [ISO speed, below](#)), the ideal pixel levels would be 236, 195, 157, 119, 83, and 52, about 3% lower than the values measured by [Bruce Lindbloom](#) (242, 201, 161, 122, 83, and 49 for the Green channel) and provided with a Colorchecker purchased in October 2005 (243, 200, 160, 122, 85, 52). On the average, these measured values fit the equation,

$$\text{pixel level} = 255 * (10^{-\text{density}} / 1.01)^{(1/2.2)}$$

Exposure error is measured by comparing the measured levels of patches 2-5 in the bottom row (20-23 in the chart as a whole) with the [selected reference levels](#). Patches 1 and 6 (19, 24) are omitted because they frequently clip. Since pixel level is proportional to $\text{exposure}^{\text{gamma}}$, and hence $\log_{10}(\text{exposure})$ is proportional to $\log_{10}(\text{pixel level}) / \text{gamma}$ (where gamma is measured from patches 2-5), the log exposure error for an individual patch is

$$\Delta(\log \text{ exposure}) = (\log_{10}(\text{measured pixel level}) - \log_{10}(\text{reference level})) / \gamma$$

Using the mean value of $\Delta(\log \text{ exposure})$ for patches 2-5 and the equation, f-stops = 3.32 * log exposure,

$$\text{Exposure error in f-stops} = 3.32 * (\text{mean}(\log_{10}(\text{measured pixel level}) - \log_{10}(\text{reference level}))) / \gamma$$

ISO sensitivity

Starting with Imatest Master 3.5, two types of ISO sensitivity are calculated: Saturation-based Sensitivity and Standard Output Sensitivity. These measurements are described on the [ISO Sensitivity](#) page.